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Stability of Systems with Multiple Motion-Dependent Loading

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I. Introduction

THE various problems of the stability of elastic bodies subjected to nonconservative single loading have been treated by many authors.[1–10] Problems with multiparameter loading where the applied loads have been assumed to have prescribed motion dependency have also been studied,[11,12] where load magnitudes were allowed to be independent of each other. Problems in which the applied loads were assumed to be dependent on the gradient of the motion, as well as the motion itself, were treated elsewhere.[13]

It appears that a certain aspect of load generalization, not reported in the literature, lies in allowing the applied loads to have independent motion-dependencies. To study the salient features of such a class of problems, a simple two-degree-of-freedom model is considered in this paper. The model is assumed to be subjected to two generally nonconservative forces with independent motion-dependency parameters. The stability investigation is carried out and results in some interesting, and seemingly new, phenomena regarding the stability characteristics. The notion has also been extended to the study of coupled structural systems under this type of loading.[14]

II. Problem Formulation and Solution

Consider Fig. 1, which is a two-degree-of-freedom model of a column acted upon by two simultaneous subtangential loadings, and in which the values of the concentrated masses and concentrated flexibilities are arbitrarily prescribed. To analyze the stability of this system, we apply the Lagrangian formulation and obtain the following linearized equations of motion

$$\begin{aligned} ml^2(3\ddot{\phi}_1+\ddot{\phi}_2)+C[2-(2-\alpha)Pl]\phi_1+C(-1+\beta\lambda)\phi_2 &= 0 \\ ml^2(\ddot{\phi}_1+\ddot{\phi}_2)-C\phi_1+[1-(1-\beta)l]\phi_2 &= 0 \end{aligned} \tag{1}$$

Let

$$\phi_i = A_i e^{\Omega t};\quad i=1,2;\quad \omega^2=\frac{ml^2}{C}\Omega^2;\quad \lambda=\frac{Pl}{C} \tag{2}$$

Upon substitution of Eq. (2) into (1), and after appropriate manipulations we obtain the following characteristic equation of the system

$$2\omega^4+[7+(\alpha-2\beta-5)\lambda]\omega^2+[(7-\alpha)(1-\beta)\lambda^2+(\alpha+3\beta-4)\lambda+1]=0 \tag{3}$$

through the utilization of known arguments on the static and kinetic criteria of stability.[10] With the help of Eq. (3) we deduce the expressions for λ_s and λ_k, the static and kinetic critical loads, respectively. They read

$$\lambda_s=\frac{-(\alpha+3\beta-4)\pm(\Delta_s)^{1/2}}{2(2-\alpha)(1-\beta)} \tag{4}$$

wherein

$$\Delta_s=\alpha^2+9\beta^2+2\alpha\beta-4\alpha-16\beta+8 \tag{5}$$

$$\lambda_k=\frac{-(3\alpha+2\beta-19)\pm(\Delta_k)^{1/2}}{(\alpha^2+4\beta^2-4\alpha\beta-2\alpha-4\beta+9)} \tag{6}$$

where

$$\Delta_k=-32\alpha^2-160\beta^2+176\alpha\beta-32\alpha+88\beta-8 \tag{7}$$

As to the question of the range of applicability of static and kinetic methods, it is observed that their respective domains of application are determined to a great extent by the signs of Δ_s and Δ_k given by Eqs. (5) and (7). Moreover, it is noticed that the inclusion of another parameter of motion dependency has added a new dimension to the space of stability parameters, and hence it has broadened the range of respective stability domains. To investigate this matter in more detail, the plots of Δ_s and Δ_k, together with their associated signs, are shown in Fig. 2. It appears from the form of the curves, that the comparison of this problem to one with an n motion-dependency parameter would lead to a stability diagram in the n-dimensional space in which the curves would be of hyperconic type sections.

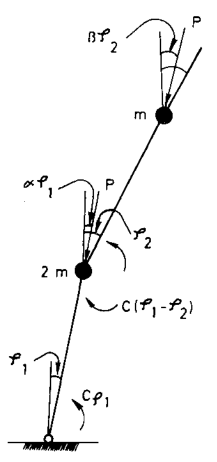

Fig. 1 Two-degrees-of-freedom model of an elastic system subjected to a multimotion dependent load.

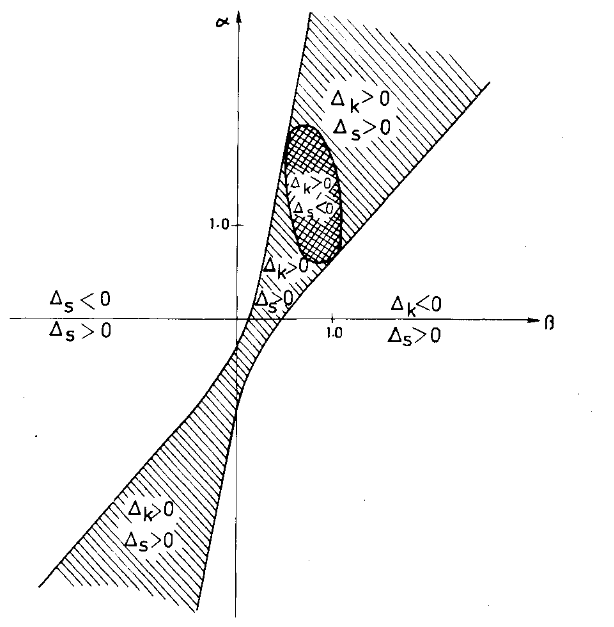

Fig. 2 Stability diagram in parameter plane.

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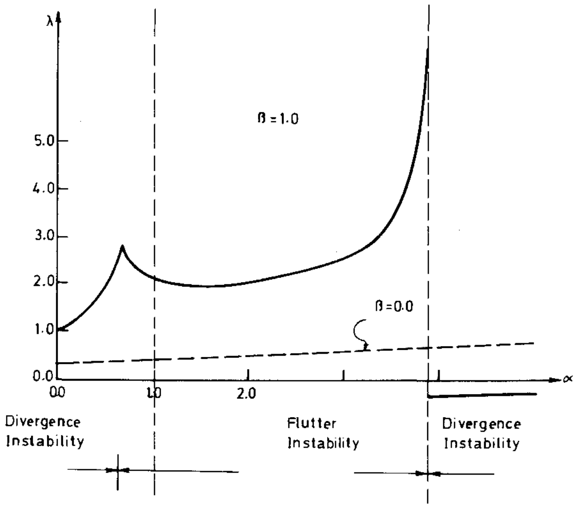

Fig. 3 Variation of critical load with parameter α.

Finally, to investigate the influence of motion-dependency parameters α and β on the magnitude of critical load, we now consider two particular cases of interest. 1) $\beta = 1$, α free to vary. The variation of critical load with parameter α, for this case is plotted in Fig. 3 (solid curves) in which the respective domain of stability criteria is also shown. As an interesting point it is observed that for the subsequent end force ($\beta = 1$), the intermediate force acting through the subtangency α can be considered as a stabilizing agent for certain values of α, shown in this figure. For the sake of comparison, the results of the particular case of constant directional vertical loading ($\beta = 0.0$), which correspond to a conservative force field, are also plotted in this figure. In the latter case the static stability analysis is applicable throughout the whole region. 2) $\alpha = 1$, β free to vary. In this case the variation of the critical force with parameter β in accordance with the static and kinetic criteria is

$$\lambda_s = \frac{3-3\beta \pm (9\beta^2)^{1/2} - 14\beta + 5}{2(1-\beta)}, \quad \beta < 0.556, \quad \beta > 1.0 \tag{8}$$

and

$$\lambda_k = \frac{8-\beta \pm \{(8-\alpha)^2 - 41[1+(1-\alpha)^2]\}^{1/2}}{2[1+(1-\alpha)^2]} \tag{9}$$

As noted, Eqs. (8) and (9) are exactly the same as the ones obtained in Ref. 4 for the case of subtangential end loading of the column. In other words, as intuitively expected, a follower state of intermediate loading would have no effect on the stability characteristics of the system.

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Boundary-Layer Transition Comparisons in Shock-Induced Flows

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Introduction

THREE methods of ascertaining boundary-layer transition in shock-induced flows are compared in this study: 1) a surface temperature technique using thin-film resistance thermometers; 2) a heat-flux technique using thin-film resistance thermometer recordings and an appropriate relation to convert from surface-temperature histories to heat-flux histories; and 3) schlieren photography of boundary-layer transition. Two shock tubes were used to generate the shock-wave induced boundary layers in order to perform the comparisons. Comparisons are made on the basis of bias introduced into the determination of transition Reynolds number.

The boundary layers in this study are those which develop in two-dimensional unsteady flow over a flat surface (shock tube wall). The governing relations for this flow are given by Mirels[1] and are, with the exception of the boundary condition of finite wall velocity, the typical[2,3] laminar boundary-layer equations. Comparisons in this study are made on the basis of transition Reynolds number as given in Ref. 1. The transition distances or times used to determine transition Reynolds numbers in this study were taken as those conventionally obtained from each of the three techniques for ascertaining transition.

Shock-Tube Facilities

The equipment used in this study consisted of two cold-gas-driven shock-tube facilities, thin-film resistance thermometers with their associated instrumentation and a schlieren photographic system.

Data for comparison of the surface-temperature technique with the heat-flux technique were obtained from a shock tube

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